Int. J Heat Mass Transfer. Vol. 16, pp. 1565–1575. Pergamon Press 1973. Printed in Great Britain

TEMPERATURE AND EDDY DIFFUSIVITY PROFILES IN NaK

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(*Received* 14 *September* 1971 *and in revised form* 29 *August* 1972)

Abstract—Local heat transfer coefficients and fully developed temperature profiles were measured in NaK eutectic mixture in a pipe at uniform wall temperature. Reynolds numbers ranged from 26 000 to 302 000 and flow was fully developed. Consistency of data was affirmed by three independent heat rate measurements. Eddy diffusivity profiles were used to calculate Nusselt numbers in pipes at uniform heat flux. Results for liquid metals are correlated by:

$$Nu(x) = Nu_\infty\left(1 + \frac{2}{x/D}\right) \qquad x/D > 4$$

$$Nu_{ave} = Nu_\infty\left(1 + \frac{8}{L/D} + \frac{2}{L/D}\ln\frac{L/D}{4}\right) \quad L/D > 4$$

where for uniform wall temperature

$$Nu_\infty = 4{\cdot}8 + 0{\cdot}0156\,Pe^{0{\cdot}85}\,Pr^{0{\cdot}08}, \qquad 0{\cdot}004 < Pr < 0{\cdot}1$$

and for uniform wall heat flux

$$Nu_\infty = 6{\cdot}3 + 0{\cdot}0167\,Pe^{0{\cdot}85}\,Pr^{0{\cdot}08}, \qquad 0{\cdot}004 < Pr < 0{\cdot}1.$$

NOMENCLATURE

D, pipe diameter;
f_m, Moody friction factor;
h, velocity defect function, equation (4);
k, thermal conductivity;
L, length of pipe;
Nu, local Nusselt number;
Nu_∞, fully developed Nusselt number;
Nu_{ave}, average Nusselt number of a length of pipe L;
Pe, Péclét number ($RePr$);
Pr, Prandtl number;
q, local heat rate;
Re, Reynolds number, Du_{ave}/ν;
r, radial coordinate;
r_0, pipe radius;
r^*, r/r_0;
T, temperature;
T_{mm}, mixed mean or bulk temperature;
T_0, temperature at $x = 0$;
T_w, inside wall temperature;
u, local mean velocity;
u_{ave}, bulk average velocity;
u_{max}, maximum velocity;
u^*, friction velocity, $\sqrt{(\tau_w/\rho)}$;
u^+, u/u^*;
x, axial coordinate;
y, $r_0 - r$, distance from pipe wall;
y^+, yu^*/ν;
ε, eddy viscosity;
ε_H, eddy diffusivity;
λ_0^2, constant in equation (1);
ρ, fluid density;
τ_w, shear stress at wall;
ν, kinematic viscosity.

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INTRODUCTION

THIS study was motivated by two considerations. First, there appear to be no unambiguous data in the literature on heat transfer to a liquid metal in a pipe at uniform wall temperature. In contrast, there have been many studies of liquid metal heat transfer in a pipe at uniform wall heat flux, though these studies show conflicting results. For fluids other than liquid metals the difference between these two temperature boundary conditions, as expressed by the Nusselt number, is small, whereas for liquid metals the difference can exceed 30 per cent [1, 2]. Second, there is considerable discrepancy in the literature concerning the eddy diffusivity in liquid metals. The purpose of this study, then, is to enlarge our knowledge of these two problems by a careful experimental study of wall heat flux and temperature distribution in NaK in fully-developed turbulent flow in a pipe at uniform wall temperature.

APPARATUS

Flow loop

Here we describe briefly the essential features of the experimental apparatus, details of which can be found in a PhD dissertation [3]. The basic apparatus was a heat transfer loop constructed of $1\frac{1}{2}$ in. schedule 10, 304 stainless steel. The loop contained a canned centrifugal pump (80 gal/min at 50 ft head), a water-cooled heat exchanger for removing heat added in the test section, a cold trap, bellows-sealed control valves, a flow meter, a hydrodynamic entrance section, the test section (horizontal), and a mixing section immediately downstream of the test section. The operating fluid was eutectic NaK, and the entire loop was lagged.

The hydrodynamic entrance section consisted of a calming chamber 3 in. dia and 6 in. long packed with flow straighteners (copper tubing) followed by a conical convergence to $1\frac{1}{8}$ in. dia. At that point a $\frac{3}{16}$ in. mesh screen was placed across the flow to initiate turbulence and thereby assure rapid achievement of fully-developed pipe turbulence. The screen was followed by 60 diameters of $1\frac{1}{8}$ in. i.d. copper tube. The inside wall of this tube (and the test section) was honed to a surface roughness of about 12 μin. Pressure taps were located 25 and 55 diameters downstream from the calming chamber, and a third pressure tap was located in the test section 1 in. from its downstream end (104 diameters from the calming chamber).

The test section was a 54-in. long tube with an i.d. of $1\frac{1}{8}$ in. ($L/D = 48$). It had a composite wall of three layers. The inner wall was of copper, 0·124 in. thick, which was metallurigically bonded to a 0·098 in. layer of constantan, which in turn was bonded to a 0·03 in. layer of brass. Ultrasonic tests of the section showed absence of any defects anywhere in either of the bonds.

Two calorimeters at $x/D = 44$ were constructed as follows. The brass and constantan layers were sliced around the tube into a ring by machining pairs of cuts 0·02 in. wide, 0·015 in. deep, and 1 in. apart. The ring was then cut longitudinally by two diametrically opposed cuts $\frac{1}{8}$ in. wide and deep enough just to penetrate the copper–constantan bond. The pads formed by these cuts served as thermocouple elements. To the layers of each pad were attached wires of copper, constantan and brass, and these wires were fabricated from the same melt of copper, constantan and brass from which the test section was made. In this way the internal tube wall temperature could be measured at two depths with virtually no disturbance to the wall temperature distribution. Thus accurate, local values of the inside tube wall temperature and of the heat flux could be obtained at this position. Smaller calorimeters were located at other axial positions, but their operation was unreliable and could not be verified, as could the 1 in. calorimeter, by temperature measurements within the liquid.

The test section was surrounded by a 3-in. stainless steel pipe which served as a steam jacket. To avoid a condensate film and consequent non-uniform wall temperature, the steam entered tangentially through a $\frac{1}{2}$-in. pipe at one end of the jacket and left through a tangential exit at the other end. The steam velocity was kept high enough to sweep out the condensate in the form of a mist, and the fraction of steam condensed in the test section was small.

Avoidance of oxides

Before filling the loop with NaK, it was evacuated, filled with argon, evacuated again, and held at a pressure of about 1 mm of mercury for 7 days. While under vacuum it was filled with NaK, and the top of the loop was then kept under a gauge pressure of about 1-in. mercury. Pressure was maintained with argon, which was introduced into the system by bubbling it through a 4-ft column loosely packed with stainless steel turnings and filled with NaK.

The cold trap was a piece of 8-in. pipe, 18 in. long packed with stainless steel wire mesh. Before a run the NaK was circulated through the trap at room temperature, and during a run the trap was by-passed.

Instrumentation

The NaK flow rate was measured by sensing with a strain gauge the drag on a disc suspended at the center of the pipe on a small rod. The instrument was calibrated with water, and the appropriate density correction was made for use in NaK.

Pressure differences between the three pressure taps were made with two differential pressure transducers of the strain gauge type.

The flow rate of cooling water to the heat exchanger was measured with an orifice, and the entering and exit water temperatures were measured with thermocouples.

All thermocouples were calibrated in place against an ice bath by circulating NaK through the loop under isothermal conditions. The outputs from the thermocouples were amplified and sent to a voltage-to-frequency converter and counter, where temperature signals could be integrated over a wide range of times, generally 20 s. This system permitted temperature to be read with a least count of 0·01°F, though the precision of measurements is of order 0·05°F.

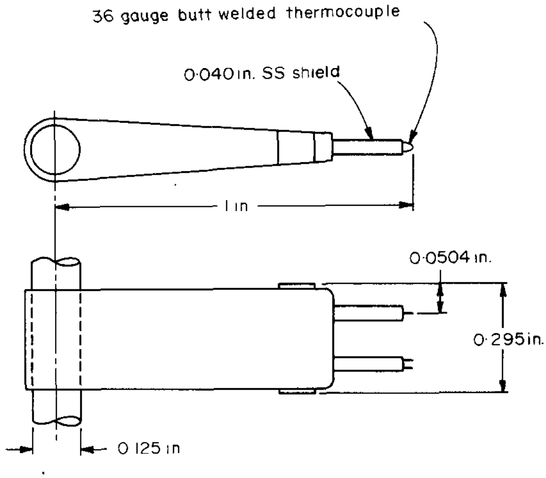

FIG. 1. Temperature probe.

The temperature distribution in the NaK was measured with a double probe illustrated in Fig. 1. The probe was located immediately downstream of the calorimeter ring on the test section. Its tip protruded upstream into the ring so that at this location, 44 diameters downstream from the thermal entrance, the wall temperature and wall heat flux as well as the temperature profile in the NaK could be measured.

The upper probe tip was a butt-welded copper–constantan thermocouple of 0·003-in. dia wire with the weld at the center of the arc of wire. The lower probe was formed by a piece of bare copper wire and bare constantan wire each 0·05-in. long and 0·007-in. apart. It was hoped that the NaK would complete the circuit of the thermocouple, but this couple gave erratic readings and was not used. The reference junction for the probe was a thermocouple well in the mixing device at the end of the test section.

ANALYSIS OF DATA

When a fluid is in fully-developed turbulent flow in a pipe of uniform wall temperature, and when the temperature profile is also fully-developed, then an eddy diffusivity can be calculated from equations which apply to this case [1]:

$$1 + \frac{Pr\varepsilon_H}{\nu} = \frac{\lambda_0^2}{2\left(\frac{r}{r_0}\right)\frac{\partial T}{\partial r}} \int_0^{r/r_0} \frac{u}{u_{\text{ave}}}\left(\frac{r'}{r_0}\right)(T - T_w)\,\mathrm{d}\left(\frac{r'}{r_0}\right) \quad (1)$$

with λ_0^2 evaluated by applying this equation at the wall, where $r/r_0 = 1$ and $\varepsilon_H = 0$. The conditions of fully-developed flow and temperature profiles were well-satisfied, but the condition of uniform wall temperature is less certain. We believe, however, that the test section design assured a wall temperature sufficiently uniform to justify the application of equation (1).

To apply equation (1) one must know $T(r)$ and especially $(\partial T/\partial r)_{r=r_0} = -(1/k)q/A$ where q/A is the local wall heat flux. In this study q/A was determined with an estimated accuracy of about 5 per cent by means of the calorimeter at $x/D = 44$. Moreover, the heat flux agreed well with that found by extrapolating the liquid temperature distribution to the wall. One uncertainty in the flux determination was the thermal conductivity of the constantan which was, therefore, determined in a separate experiment [2], the results of which are summarized below:

T (°F)	k (Btu/hft °F)
137	12·8
150	13·0
164	13·2
178	13·4

The velocity distribution is required for use in equation (1), and values of eddy viscosity ε are needed to calculate the ratio $\varepsilon_H/\varepsilon$. Velocity distribution was not measured in these experiments, but the design of the loop assured fully-developed velocity profiles in the test section. Accordingly, u and ε were evaluated as follows: For the region near the wall, $y^+ < 45$, no calculations of ε were made.

For the outer wall (constant stress) region:

$$\frac{\varepsilon}{\nu} = 0{\cdot}4\, r^* y^+ - 1 \quad (2)$$

$$u^+ = 5{\cdot}1 + 2{\cdot}5 \ln y^+ . \quad (3)$$

$$\left.\right\} y^+ > 45 \text{ and } y/r_0 < 0{\cdot}15$$

For $y/r_0 > 0{\cdot}15$, the velocity defect law:

$$\frac{u_{\max} - u}{u^*} = h(y/r_0), \quad (4)$$

from which it follows that

$$\frac{\varepsilon}{\nu} = \frac{Re\sqrt{(f_m/8)}}{4\,\mathrm{d}h/\mathrm{d}r^{*2}} - 1, \quad (5)$$

or since ε/ν is usually $\gg 1$ for $y/r_0 > 0{\cdot}15$, equivalently

$$\frac{\varepsilon}{Du^*} = \frac{1}{4\,\mathrm{d}h/\mathrm{d}r^{*2}}. \quad (6)$$

The velocity defect law, given by equation (4) and also called the "law of the wake", has long been regarded as an empirical fact. For example, von Kármán [4] established the function h from data of Nikuradse and gave a semi-empirical equation which fit the data fairly well. (The line shown on his velocity defect plot is apparently a line through the data, not a plot of his equation.) Since ε is given by equation (5), it is necessary only to have h in empirical form to determine ε. The values of h and $\mathrm{d}h/\mathrm{d}r^{*2}$ used here were determined from the data of Nikuradse [5], Laufer [6], and Sleicher [7], which differ very little from each other, and are given in Table 1. Note that this empirical table shows ε to be

Table 1. Velocity defect function

y/r_0	h	$\mathrm{d}h/\mathrm{d}r^{*2}$
0·15	5·6	9·7
0·2	4·8	8·8
0·3	3·55	7·65
0·4	2·6	7·3
0·5	1·8	7·2
0·6	1·15	7·2
0·7	0·65	7·2
0·8	0·3	7·2
0·9	0·07	7·2

constant over the inner 50 per cent of the pipe radius, where $h = 7{\cdot}2\ r^{*2}$. The coefficient 7·2 agrees precisely with the value found by Hinze [8] for the data of Laufer. Inspection of Hinze's Fig. 7–49 shows that the assumption of a constant value of ε for $r^* < 0{\cdot}5$ is a nearly perfect fit to the data. Hence for the inner 50 per cent of the pipe radius, equation (6) reduces to

$$\frac{\varepsilon}{Du^*} = \frac{1}{4(7{\cdot}2)} = 0{\cdot}035, \qquad r^* < 0{\cdot}5, \quad (7)$$

as found earlier by a different but equivalent route by Hinze [8]. For an interesting discussion of eddy diffusivity in the core of pipe flow and for further evidence that eddy diffusivity is essentially constant in the core region, the reader is referred to a paper by Brinkworth and Smith [9].

RESULTS AND DISCUSSIONS

Six runs were made over a Reynolds number range of 26 000–302 000, inlet NaK temperatures ranged from 122°F to 177°F, and bulk to wall temperature difference at the temperature probe ranged from 1·6 to 12°F. The Prandtl numbers ranged from 0·0203 to 0·0245. The experimental conditions were such that the criteria established by Buhr, Carr and Balzhizer [10] indicate that free convection effects were negligible.

For each run friction factors were calculated from pressure drop measurements between the two taps at 55 and 104 diameters from the entrance, which encompass the test section. Similar measurements were also made with iso-octane during testing of the loop. In all cases the friction factors were within 4 per cent of Moody chart values and hence confirm that the test section was hydraulically smooth. The friction factors for the NaK runs are given in Table 2.

Figure 2 shows a temperature profile for an intermediate value of Reynolds number. Some scatter of the data is evident. The scatter is caused by the statistical nature of the temperature and the finite averaging time, 20 s. A smooth line was drawn through the data, and points on the line were used to determine dT/dr by a least squares fit of a fourth-degree polynomial to seven points (Douglass–Avakian method).

Data for each run are given in Table 2. The temperature distributions are smoothed data

Table 2. Temperature profiles

Re		26 000	52 000	79 000	106 000	203 000	302 000
f_m		0·0245	0·021	0·0185	0·017	0·0155	0·015
Pr		0·0203	0·0212	0·0219	0·0225	0·0235	0·0245
T_{mm}(°F)		198·06	186·80	176·15	167·05	152·6	137·2
q/A (Btu/h ft^2)		1490	4640	9670	15 150	25 800	45 500
Nu_∞		6·8	9·1	10·5	13·5	20	29
T(°F)	0	196·48	183·45	170·6	160·75	146·3	129·5
at values	0·1	196·53	183·55	170·75	160·9	146·6	129·75
of r^*	0·2	196·68	183·8	171·1	161·35	147·1	130·3
	0·3	196·90	184·2	171·8	162·15	147·75	131·3
	0·4	197·18	184·8	172·65	163·15	148·8	132·6
	0·5	197·50	185·55	173·9	164·55	150·0	133·95
	0·6	197·85	186·3	175·15	165·9	151·25	135·5
	0·7	198·25	187·15	176·6	167·5	152·9	137·4
	0·75	198·45	187·6	177·35	168·4	153·4	138·4
	0·8	198·68	188·05	178·15	169·3	154·85	139·6
	0·85	198·90	188·6	179·1	170·3	155·85	140·9
	0·9	199·13	189·1	180·15	171·5	157·25	142·5
	0·95	199·39	189·95	181·5	173·0	159·15	144·7
	1	199·67	190·6	183·05	175·4	162·7	149·5

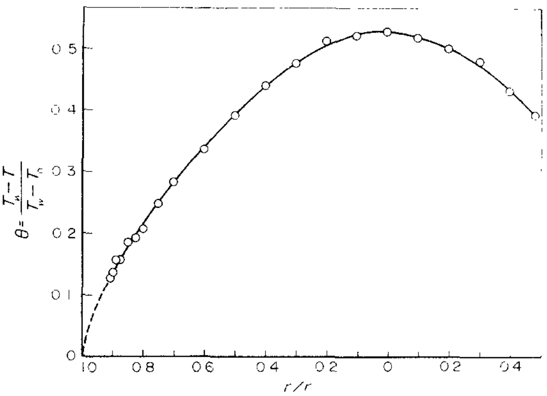

FIG. 2. Temperature profile at Re = 106 000.

and are shown to the nearest 0·1°F except for Re = 26 000, where the small wall-to-bulk temperature difference resulted in small temperature fluctuations and, hence, more precise values of the time-averaged temperature. T_{mm} was determined in two ways—by integration of the temperature profile and by its measurement in the mixing chamber. The two agreed quite closely, but the mixing chamber temperature was always slightly greater than the temperature from profile integration—probably because of some heat transfer downstream of the last calorimeter. Therefore, the most accurate values of T_{mm} are calculated from the temperature profiles, and these were used to determine Nu at x/D = 44.

Figure 3 shows the eddy diffusivity ratio as a function of radius for each of the runs. Also shown is a diffusivity ratio profile reported by Brown, Amstead and Short [11] for flow of mercury (Pr = 0·02, about the same as ours) in a pipe at uniform heat flux. Only their lowest Reynolds number (250 000) is shown because the relative roughness of their test section, 0·00015, means that at higher Reynolds numbers their test section was not hydraulically smooth. Hence their results should not be expected to be comparable to ours. At Re = 250 000 their results and ours are not greatly different, but both of these results differ substantially from the profiles reported by Isakoff and Drew [12]. This disagreement and the fact that the Nusselt numbers found by Isakoff and Drew exceed those reported in most other studies suggest that profiles reported by Isakoff and Drew are not representative of fully-developed flow of liquid metals in smooth pipes.

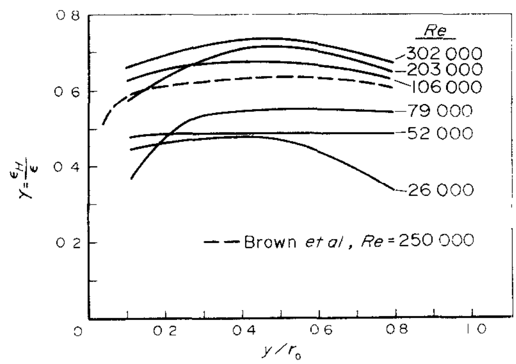

FIG. 3. Diffusivity ratio profiles.

The profiles shown also disagree with those reported by Buhr, Carr and Balzhiser [10]. Part of the disagreement is caused by their use of an eddy diffusivity which decreases to a relatively small value at the pipe center. As discussed earlier, it is our belief that such a profile is suspect and that the rise in their values of $\varepsilon_H/\varepsilon$ for $y/r_0 > 0{\cdot}5$ is caused in part by their choice of ε. Their $\varepsilon_H/\varepsilon$ profiles also show a much greater effect of Reynolds number than do ours. In particular, their profiles at Re = 200 000 and Re = 300 000 greatly exceed ours at every point. The cause of this discrepancy is not certain, but a possible explanation is the swirl flow (axial component of vorticity) that existed in their test section. The liquid metal entered their test section through the side of a pipe Tee, which produced a swirl component of flow immediately downstream [13]. Their temperature profiles were measured at 74 and 105 diameters downstream, but this is an insufficient distance to reduce swirl to a small fraction of its initial value. Rochino and Lavan [14] analyzed turbulent swirl flow in a pipe, and their analysis and data show that swirl decays at a rather low rate, that the rate of decay decreases with increasing

Reynolds number, and that at Re of order 250 000, the tangential component of velocity is attenuated by roughly 50 per cent at $x/D = 100$. Therefore, swirl flow could be responsible for magnifying the values of $\varepsilon_H/\varepsilon$ and of the Nusselt number, especially at high Reynolds numbers, in the experiments of Buhr, Carr and Balzhiser.

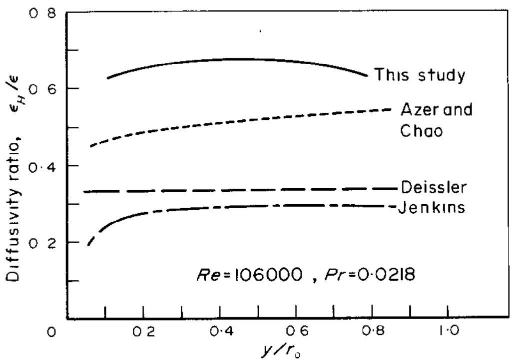

FIG. 4. Comparison of diffusivity ratios with theoretical models at $Re = 106\,000$.

Figure 4 shows a comparison of the results at an intermediate Reynolds number with the model equations of Jenkins [15], Deissler [16] and Azer and Chao [17]. It is clear that the equation of Azer and Chao fits the data best, and it does so at other Reynolds numbers as well.

The Nusselt numbers at $x/D = 44$ are all fully-developed and are shown in Fig. 5 together

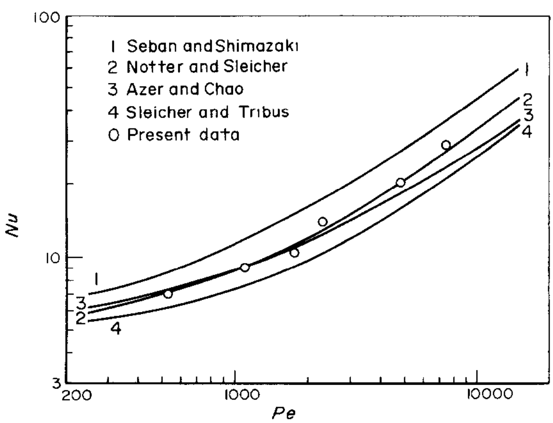

FIG. 5. Comparison of fully developed Nusselt numbers with proposed correlations.

with the following correlation equations proposed for liquid metal heat transfer in a pipe of uniform wall temperature

$$Nu_\infty = 5 + 0{\cdot}025\, Pe^{0{\cdot}8}$$

Seban and Shimazaki [18] (8)

$$Nu_\infty = 4{\cdot}8 + 0{\cdot}015\, Pe^{0{\cdot}91}\, Pr^{0{\cdot}30}$$

Sleicher and Tribus [1] (9)

$$Nu_\infty = 5 + 0{\cdot}05\, Pe^{0{\cdot}77}\, Pr^{0{\cdot}25}$$

Azer and Chao [19] (10)

$$Nu_\infty = 4{\cdot}8 + 0{\cdot}0156\, Pe^{0{\cdot}85}\, Pr^{0{\cdot}08}$$

Notter and Sleicher [20] (11)

These four equations are all empirical fits to calculations, which differ from author to author primarily in the eddy diffusivity distribution assumed. Seban and Simazaki took $\varepsilon_H = \varepsilon$; Sleicher and Tribus used a modification of the eddy diffusivity model of Jenkins [15], and Azer and Chao used their own eddy diffusivity model, which was fit to considerable data. Notter and Sleicher [20], used the data of Awad [3], as reanalyzed and reported here, to determine their eddy diffusivities. Hence their equation is essentially a fit to the data reported here, and the equation fits the data within 10 per cent.

Consistency with data at uniform wall heat flux

Since the temperature field in turbulent flow is a function of the temperature boundary conditions, evidently the eddy diffusivity must also depend upon the temperature or heat flux distribution at the boundary. It is known, in fact, that the value of eddy diffusivity at a point depends upon the entire history of the temperature and velocity fields. Yet for the case of fully developed pipe flow, intuition based on models such as various mixing length theories suggests that the dependence upon the wall temperature boundary condition is weak. Evidence for this suggestion is found in the apparent success of thermal entry length calculations based on the assumption that the eddy diffusivity is independent of the temperature boundary conditions. Hence the eddy diffusivity found here can be used to calculate Nusselt numbers for a pipe at constant wall heat flux. This has been done by Notter and Sleicher [20], who report that the equation

$$Nu_\infty = 6{\cdot}3 + 0{\cdot}0167\, Pe^{0{\cdot}85}\, Pr^{0{\cdot}08} \qquad 0{\cdot}004 < Pr < 0{\cdot}1 \qquad (12)$$

correlates their calculations and is a good fit to data in the absence of entry length and probe disturbance effects. We note here that their entry length calculations in the thermal entry region at both uniform wall temperature and wall flux in the liquid metal region are correlated within about 20 per cent by

$$Nu = Nu_\infty\left(1 + \frac{2}{x/D}\right), \qquad \frac{x}{D} > 4, \quad (13)$$

where Nu_∞ is found from equations (11) or (12). For these cases, the average Nusselt number for $x/D < 4$ is very roughly 3 Nu_∞, which together with equation (13) gives the average Nusselt number for a pipe of length L/D

$$Nu_{\text{ave}} = Nu_\infty\left[1 + \frac{8}{L/D} + \frac{2}{L/D}\ln\frac{L/D}{4}\right], \qquad \frac{L}{D} > 4. \quad (14)$$

Reliability of results

In interpreting results it should be kept in mind that the fluid temperatures reported are estimated mean temperatures of a random variable determined by averaging over 20 s, and therefore scatter of the data is larger than the least count of our instruments, 0·01°F. Two internal consistency tests can be applied to the data. First, the inside wall temperature at $x/D = 44$ was measured both by extrapolation from the internal tube wall temperatures and by extrapolation of the fluid temperature. Extrapolation from the fluid side was done with

$$T_w = T_{0{\cdot}90} + 0{\cdot}02\left[\left(\frac{\mathrm{d}T}{\mathrm{d}r^*}\right)_{0{\cdot}91} + \left(\frac{\mathrm{d}T}{\mathrm{d}r^*}\right)_{0{\cdot}93} + \left(\frac{\mathrm{d}T}{\mathrm{d}r^*}\right)_{0{\cdot}95} + \left(\frac{\mathrm{d}T}{\mathrm{d}r^*}\right)_{0{\cdot}97} + \left(\frac{\mathrm{d}T}{\mathrm{d}r^*}\right)_{0{\cdot}99}\right]$$

where the subscripts denote values of r^*. The derivatives were calculated from equation (1) with the assumption that $1 + Pr\,\varepsilon_H/\nu$ is linear in the range $0{\cdot}9 < r^* < 1$. This procedure leads to little error with liquid metals. Table 3 shows the results of the wall temperature measurements together with the values of T_{mm}. The difference between the two determinations of wall temperature divided by $T_w - T_{\text{mm}}$ gives an indication of the per cent difference in the Nusselt number that would be attributed to the differences in T_w, and this ratio is also given in the table.

A second way in which the internal consistency of the data can be evaluated is provided by heat balances. Table 4 shows the heat rate to the apparatus determined by three independent methods. The maximum deviation among the 12 possible pair comparisons is 7 per cent. Some of the systematic differences in the heat balances have plausible explanations: $B > A$ because of

Table 3. Wall and mixed mean fluid temperatures

Re	T_{ww}	T_{wf}	T_{mm}	$(T_{wf} - T_{ww})/(T_{wf} - T_{\text{mm}})$, %
26 000	199·59	199·67	198·06	5
52 000	190·34	190·58	186·80	6
79 000	182·59	183·05	176·15	7
106 000	175·38	175·38	167·05	0
203 000	161·88	162·67	152·6	8
302 000	148·73	149·52	137·2	6

Temperatures in °F; T_{ww} measured from wall side, T_{wf} from fluid side.

Table 4. Heat balances
(Heat rate in Btu/h × 10^{-3})

	$Re \times 10^{-3}$					
	26	52	79	106	203	302
A	10·4	15·9	30·7	43	60	101
B	10·5	16·2	32·4	44	63	101
C	9·8	15·9	31·5	43	60	95
Deviation (%)	1–7	0–2	3–5	0–2	0–5	0–6

A from $T_{mm} - T_0$, T_{mm} from profile integration
B from $T_{mm} - T_0$, T_{mm} from mixing chamber
C from cooling water

some heat transfer downstream of the probe, and C may be low because some heat is lost through the lagging of the apparatus which, then, does not appear in the cooling water. We believe that the heat balances provide strong evidence of the validity of the data.

Finally, the fact that the uniform wall temperature Nusselt numbers found here have been shown to be consistent with the uniform wall flux data of other authors is another indication of the validity of the results. The uniform flux Nusselt number calculations and internal consistency checks discussed above imply that the experimental data give reliable temperature and eddy diffusivity profiles for turbulently flowing NaK at uniform wall temperature.

CLOSURE

We close this paper with two thoughts. First, the discrepancy between the present results and those of Buhr, Carr and Balzhiser [10] suggests that swirl may augment heat transfer to liquid metals to a significant degree. A systematic investigation of this subject would, therefore, be interesting and potentially valuable.

Second, data on heat transfer to liquid metals are difficult to obtain, and it is all the more important, therefore, to be able to assess the reliability and meaning of the results. Towards this end, experiments should be designed to yield redundant measurements so that internal consistency checks are possible.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of the National Science Foundation for this study.

REFERENCES

1. C. A. Sleicher, Jr. and M. Tribus, Heat transfer in a pipe with turbulent flow and arbitrary wall-temperature distribution, *Trans. Am. Soc. Mech. Engrs* **79**, 789–797 (1957).
2. R. H. Notter, Two problems in turbulence A. A theoretical and empirical study of the limiting form of the eddy diffusivity near a wall. B. Matched asymptotic expansions applied to the turbulent Graetz problem, PhD dissertation, University of Washington, Seattle (1969).
3. A. S. Awad, Heat transfer and eddy diffusivity in NaK in a pipe at uniform wall temperature, PhD dissertation, University of Washington, Seattle (1965).
4. Th. von Kármán, Turbulence and skin friction, *J. Aeronaut. Sci.* **1**, 1–20 (1934).
5. J. Nikuradse, Gestzmàssigkeiten der turbulenten Strömung in glatten Röhren, *ForschHft. Ver. Dt. Ing.* 356 (1932).
6. J. Laufer, The structure of turbulence in fully developed pipe flow, NACA TR 1174 (1954).
7. C. A. Sleicher, Experimental velocity and temperature profiles for air in turbulent pipe flow, *Trans. Am. Soc. Mech. Engrs* **80**, 693–703 (1958).
8. J. O. Hinze, *Turbulence*, p. 536. McGraw-Hill, New York (1959).
9. B. J. Brinkworth and P. C. Smith, Velocity distribution in the core of turbulent pipe flow, *Chem. Engng Sci.* **24**, 787–791 (1969).
10. H. O. Buhr, A. D. Carr and R. E. Balzhiser, Temperature profiles in liquid metals and the effect of superimposed free convection in turbulent flow, *Int. J. Heat Mass Transfer* **11**, 641–654 (1968).
11. H. E. Brown, B. H. Amstead and B. E. Short, Temperature and velocity distribution and transfer of heat in a liquid metal, *Trans. Am. Soc. Mech. Engrs* **79**, 279–285 (1957).

12. S. E. ISAKOFF and T. B. DREW, Heat and momentum transfer in turbulent flow of mercury, General Discussion on Heat Transfer, pp. 405–409. Institution of Mechanical Engineers, London (1951).
13. M. J. TUNSTALL and J. K. HARVEY, On the effect of a sharp bend in a fully developed turbulent pipe-flow, *J. Fluid Mech.* **34**, 595–608 (1968).
14. A. P. ROCHINO and Z. LAVAN, Analytical investigation of incompressible turbulent swirling flow in pipes, NASA CR-1169, 128 p. (1968).
15. R. JENKINS, Variation of eddy conductivity with Prandtl modulus and its use in prediction of turbulent heat transfer coefficients, *1951. Heat Transfer and Fluid Mechanics Institute*, pp. 147–158. Stanford University Press, Stanford (1951).
16. R. G. DEISSLER, Analysis of fully developed turbulent heat transfer at low Péclét numbers in smooth tubes with applications to liquid metals, NACA RM E 52F05 (1952).
17. N. Z. AZER and B. T. CHAO, A mechanism of turbulent heat transfer in liquid metals, *Int. J. Heat Mass Transfer* **1**, 121–138 (1960).
18. R. A. SEBAN and T. T. SHIMAZAKI, Heat transfer to a fluid flowing turbulently in a smooth pipe with walls at constant temperature, *Trans. Am. Soc. Mech. Engrs* **73**, 803–808 (1951).
19. N. Z. AZER and B. T. CHAO, Turbulent heat transfer in liquid metals—fully developed pipe flow with constant wall temperature, *Int. J. Heat Mass Transfer* **3**, 77–83 (1961).
20. R. H. NOTTER and C. A. SLEICHER, A solution to the turbulent Graetz problem—III. Fully developed and entry region heat transfer rates, *Chem. Engng Sci.* **27** 2073–2093 (1972).

PROFILS DE TEMPERATURE ET DE DIFFUSIVITE PAR TURBULENCE DANS DU NaK

Résumé—Des coefficients locaux de transfert thermique et des profils de température établis ont été mesurés dans un mélange eutectique de NaK dans un tuyau à température pariétale uniforme Les nombres de Reynolds varient entre 26000 et 302000 et l'écoulement est entièrement établi. La qualité des résultats a été confirmée par trois mesures indépendantes du flux thermique. Les profils de diffusivité par turbulence ont été utilisés pour le calcul des nombres de Nusselt dans des tuyaux avec flux thermique uniforme. Les résultats pour les métaux liquides sont représentés par,

$$Nu(x) = Nu_{\infty}\left(1 + \frac{2}{x/D}\right) \quad x/D > 4$$

$$Nu_{ave} = Nu_{\infty}\left(1 + \frac{8}{L/D} + \frac{2}{L/D}\ln\frac{L/D}{4}\right) \quad L/D > 4$$

où pour une température pariétale uniforme

$$Nu_{\infty} = 4{,}8 + 0{,}0156 Pe^{0{,}85} Pr^{0{,}08} \quad 0{,}004 < Pr < 0{,}1$$

et pour un flux thermique pariétal uniforme

$$Nu_{\infty} = 6{,}3 + 0{,}0167 Pe^{0{,}85} Pr^{0{,}08} \quad 0{,}004 < Pr < 0{,}1.$$

DIE PROFILE DER TEMPERATUR UND DER TURBULENTEN AUSTAUSCHKOEFFIZIENTEN IN NaK

Zusammenfassung—In einem Rohr wurden bei gleichförmiger Wandtemperatur örtliche Wärmeübergangskoeffizienten und voll ausgebildete Temperaturprofile in der eutektischen Mischung NaK ausgemessen. Die *Re*-Zahlen lagen im Bereich von 26000 bis 302000 und die Strömung war voll eingelaufen. Die Konsistenz der Daten wurde durch drei unabhängige Wärmestrommessungen sichergestellt. Die Profile des turbulenten Austauschkoeffizienten wurde zur Berechnung der *Nu*-Zahlen in Rohren bei konstanter Wärmestromdichte herangezogen. Die Ergebnisse für flüssige Metalle werden durch folgende Gleichung zusammengefasst:

$$Nu_{(x)} = Nu_{\infty}\left(1 + \frac{2}{x/D}\right) \quad x/D > 4$$

$$Nu_{ave} = Nu_{\infty}\left(1 + \frac{8}{L/D} + \frac{2}{L/D}\ln\frac{L/D}{4}\right) \quad L/D > 4.$$

Für gleichförmige Wandtemperatur gilt:

$$Nu_{\infty} = 4{,}8 + 0{,}0156\, Pe^{0.85}\, Pr^{0.08}, 0.004 < Pr < 0.1.$$

Für konstante Wärmestromdichte der Wand gilt:

$$Nu_{\infty} = 6{,}3 + 0{,}0167\, Pe^{0.85}\, Pr^{0.08}, 0{,}004 < Pr < 0.1.$$

ПРОФИЛИ ТЕМПЕРАТУРЫ И ТУРБУЛЕНТНОЙ ДИФФУЗИИ В NaK

Аннотация—Измерялись локальные коэффициенты теплообмена и полностью развитые профили температуры при течении эвтектической смеси NaK в трубе при однородной температуре стенки. Значения числп Рейнольдса изменялись в диапазоне 26 000–302 000, при котором течение было полностью развитым, Совпадение данных подтвердилось тремя независимыми измерениями удельного расхода тепла. Профили турбулентной диффузии использовались для расчета значений числа Нуссельта в трубе при однородном тепловом потоке. Результаты для жидких металлов обобщаются уравнениями:

$$Nu(x) = Nu_{\infty}\left(1+\frac{2}{x/D}\right) \quad x/D > 4$$

$$Nu_{\text{ave}} = Nu_{\infty}\left(1+\frac{8}{L/D}+\frac{2}{L/D}\ln\frac{L/D}{y}\right) \quad L/D > y,$$

где для однородной температуры стенки

$$Nu_{\infty} = 7{,}8+0{,}0156\, Pe^{0{,}85}\, Pr^{0{,}08}, \; 0{,}004 < Pr < 0{,}1,$$

а для однородного теплового потока

$$Nu_{\infty} = 6{,}3+0{,}0167\, Pe^{0{,}85}\, Pr^{0{,}08}, \; 0{,}004 < Pr < 0{,}1.$$